Swing wave-wave interaction: Coupling between fast magnetosonic and Alfvén waves

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We suggest a mechanism of energy transformation from fast magnetosonic waves propagating across a magnetic field to Alfvén waves propagating along the field. The mechanism is based on *swing wave-wave interaction* [T. V. Zaqarashvili, Astrophys. J. Lett. **552**, 107 (2001)]. The standing fast magnetosonic waves cause a periodical variation in the Alfvén speed, with the amplitude of an Alfvén wave being governed by Mathieu's equation. Consequently, subharmonics of Alfvén waves with a frequency half that of magnetosonic waves grow exponentially in time. It is suggested that the energy of nonelectromagnetic forces, which are able to support the magnetosonic oscillations, may be transmitted into the energy of purely magnetic oscillations. Possible astrophysical applications of the mechanism are briefly discussed.

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I. INTRODUCTION

Many observed phenomena can be associated with wave-like motions, increasing interest in the study of wave dynamics. Linear perturbation theory considers an arbitrary disturbance as a superposition of independently evolving eigenmodes, thus simplifying the description of the process. However, interactions between different harmonics as well as between different kinds of waves leads to the appearance of substantially new phenomena.

In the case of large-amplitude acoustic waves, nonlinearity leads to the generation of higher harmonics that cause steepening of the wave front and consequently the formation of shock waves. Also developments in plasma theory raised interest in the study of interactions between different waves. It is shown that nonlinear interaction leads to the generation of resonant triplets (or multiplets) in the plasma [1–3]. The nonlinear interaction between magnetohydrodynamic (MHD) waves has been studied in various astrophysical situations [4–7]. Additionally, MHD wave coupling due to inhomogeneity of the medium [8–12] or a background flow [13–15] has also been developed.

Recently, a new kind of interaction between sound and Alfvén waves has been discussed by Zaqarashvili [16]. The physical basis of this interaction is the parametric influence; sound waves cause a periodical variation in the medium's parameters, which affects the velocity of transversal Alfvén waves and leads to a resonant energy transformation into certain harmonics. In a high-β plasma, it is shown that periodical variations of the medium's density, caused by the propagation of sound waves along an applied magnetic field, results in Alfvén waves being governed by Mathieu's equation (here $\beta = 8\pi p/B^2 \gg 1$, where p is the plasma pressure and B is the magnetic field). Consequently, harmonics with half the frequency of sound waves grow exponentially in time. The same phenomenon was developed in the case of standing sound waves [17]. The process of energy exchange between these different kinds of wave motion is called *swing wave-wave interaction*. This terminology arises from an analogy with a swinging pendulum, as described below.

In this paper we further develop the theory for interactions between fast magnetosonic waves and Alfvén waves. For clarity of presentation we first recall the pendulum analogy and show that under certain conditions the energy of spring oscillations along the pendulum axis is transformed into the energy of transversal oscillations of the pendulum, and vice versa. Following a discussion of the general physics of swing interaction, we consider the example of coupling between fast magnetosonic waves propagating across an applied magnetic field and Alfvén waves propagating along the field. Finally, we briefly describe the applications of the theory to various astrophysical situations.

II. SWING PENDULUM

It is useful to begin with a mechanical analogy of the wave dynamics in a medium (see Ref. [2] in the case of three-wave interaction). Consider a mathematical pendulum with mass m and equilibrium length L (see Fig. 1). Part of the pendulum length consists of a spring with stiffness constant σ. There are two kinds of oscillations in this system,

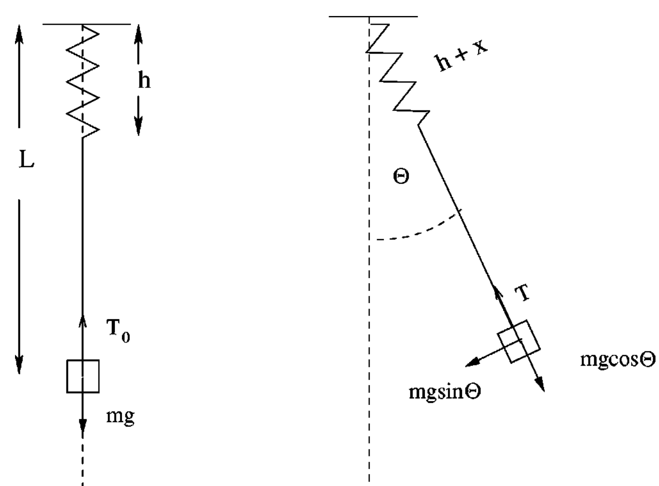

FIG. 1. The swing pendulum in equilibrium (left) and in oscillation (right).

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transversal oscillations due to gravity and spring oscillations along the pendulum axis due to the elasticity of the spring. This is a *swing pendulum*.

In equilibrium, gravity is balanced by the stiffness force T_0 of the spring so that

$$T_0=\sigma h=mg,$$

where g is the gravitational acceleration and h is the equilibrium length of the spring (the natural length of the spring is supposed to be negligible). For displacement x along the pendulum axis, the stiffness force becomes

$$T=\sigma(h+x)=mg+\sigma x.$$

Newton's second law applied along the pendulum axis, when the pendulum makes an angle Θ with the vertical (see Fig. 1), gives the equation of motion (the centrifugal force due to the transversal oscillation is neglected),

$$\ddot{x}+\frac{\sigma}{m}x=g(\cos\Theta-1).$$

Due to the oscillation of the spring along the axis, the pendulum length is a function of time and the equation of transversal motions of the pendulum under gravity is

$$\ddot{\Theta}+\frac{g}{L+x}\sin\Theta=0.$$

For clarity of presentation a term $2\dot{x}\dot{\Theta}/(L+x)$ is neglected here; it does not affect the physical nature of the phenomenon (for general consideration, see Refs. [18,19]). So we have two different oscillations of the pendulum, which are coupled, and each oscillation influences the other. Considering small amplitude oscillations, we find two coupled equations governing the dynamics of the pendulum:

$$\ddot{x}+\omega_1^2x=-\frac{1}{2}g\Theta^2, \tag{1}$$

$$\ddot{\Theta}+\omega_2^2\left(1-\frac{x}{L}\right)\Theta=0, \tag{2}$$

where $\omega_1=\sqrt{g/h}$ and $\omega_2=\sqrt{g/L}$ are the fundamental frequencies of the system.

From Eqs. (1) and (2) we can see that longitudinal oscillations of the pendulum causes a periodical variation of the pendulum length. In certain conditions this can lead to the well-known parametric amplification of transversal oscillations. When x is a periodical function of time, then Eq. (2) becames Mathieu's equation and it has a resonant solution when

$$\omega_2=\frac{1}{2}\omega_1, \tag{3}$$

corresponding to $L=4h$.

Under these conditions, initial spring oscillations x along the pendulum axis can amplify small transversal perturbations Θ [see Eq. (2)]. On the other hand, transversal oscillations may be considered as an external periodic force [see Eq. (1)] that causes the damping and consequent amplification of longitudinal oscillations. So, in the absence of dissipation, there is a subsequent energy exchange between different oscillations in the system. But if some kind of external force supports the spring oscillations then they can amplify the transversal oscillations until nonlinear effects became significant.

III. SWING WAVE-WAVE INTERACTION

The generalization of the above analogy to waves in a medium leads to interesting phenomena. Spring oscillations do work against gravity and cause periodical variations of the parameter (pendulum length L) of transversal oscillations. As a result of this work, the energy of spring oscillations transforms into the energy of transversal oscillations. So we may expect a similar process in a medium when one kind of wave causes a periodical variation of another wave parameter.

There are three main forces in the equation of motion for an ideal conductive fluid: the pressure gradient $-\nabla p$, gravity $\rho\nabla\phi$, and the Lorentz force $\mathbf{j}\times\mathbf{B}$. Here p and ρ denote the plasma pressure and density, ϕ is the gravitational potential, and $\mathbf{j}$ is the current in a magnetic field $\mathbf{B}$. Each of these forces represents the restoring force against the fluid inertia and thus leads to the generation of different kinds of wave motions. Of these forces, only the Lorentz force does not include the density (in the pressure gradient the density arises from the equation of state). This fact leads to the appearance of the density in the expression for the magnetic speed, the Alfvén speed $V_A=B/\sqrt{4\pi\rho}$, which describes the propagation of magnetic waves and depends on the medium density ρ. For a similar reason, the frequency of pendulum oscillations does not depend on the pendulum mass (because the gravitational force depends on it), while the frequency of spring oscillation does depend on the mass (because the stiffness force does not depend on it). On the other hand, compressible waves cause density variations in the medium and therefore they may affect the propagation properties of magnetic waves. This suggests a coupling between longitudinal, compressible waves (leading to density perturbations) and transversal magnetic waves propagating with a velocity that depends on the density. The latter can be associated with Alfvén waves that are transversal and represents the purely electromagnetic properties of the medium. The compressible waves cause a periodical variation of the density and so of the Alfvén speed, and may lead to the effective energy transmission into certain harmonics of Alfvén waves. The swing coupling between sound and Alfvén waves propagating along an applied magnetic field [16,17] is a good example of this phenomenon. On the other hand, magnetosonic waves propagating at an angle to the magnetic field also cause periodical variations of the Alfvén speed and may lead to similar phenomena. It is worth noticing that, contrary to the Alfvén waves, the magnetosonic waves can be easily excited in a medium by any force (even of nonelectromagnetic origin). Therefore, the coupling between magnetosonic and Alfvén waves allows the transmission of energy into purely transversal magnetic oscillations through compressible magnetosonic oscillations.

To show the mathematical formalism of swing wave interaction we consider the case of magnetosonic wave propagation across the applied magnetic field. In this case we have fast magnetosonic waves. For simplicity we consider a rectangular geometry, that then can be generalized to cylindrical and spherical symmetries.

Coupling between fast magnetosonic and Alfvén waves

Consider motions of a homogeneous medium, with zero viscosity and infinite conductivity, as described by the ideal MHD equations:

$$\frac{\partial \mathbf{B}}{\partial t}+(\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{B}=(\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{u}-\mathbf{B}\boldsymbol{\nabla}\cdot\mathbf{u}, \quad \boldsymbol{\nabla}\cdot\mathbf{B}=0, \tag{4}$$

$$\rho\frac{\partial \mathbf{u}}{\partial t}+\rho(\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}=-\boldsymbol{\nabla}\left[p+\frac{B^2}{8\pi}\right]+\frac{(\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B}}{4\pi}, \tag{5}$$

$$\frac{\partial \rho}{\partial t}+(\mathbf{u}\cdot\boldsymbol{\nabla})\rho+\rho\boldsymbol{\nabla}\cdot\mathbf{u}=0, \tag{6}$$

where $\mathbf{u}$ is the fluid velocity. We consider adiabatic processes, so the pressure p and density ρ are connected by the relation

$$p=p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}, \tag{7}$$

where p_0 and ρ_0 are the unperturbed uniform pressure and density and γ is the ratio of specific heats. We neglect gravity, though it may be of importance under some astrophysical conditions.

Linear analysis of Eqs. (4)–(7) show the existence of three kinds of MHD waves: Alfvén and magnetosonic (fast and slow) waves. The difference between these waves is that the restoring force of Alfvén waves is the tension of magnetic field lines $(\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{B}/4\pi$, acting alone, while the restoring force of magnetosonic waves is mainly the gradient of ordinary and magnetic pressures, $-\boldsymbol{\nabla}[p+B^2/8\pi]$. The various waves can be distinguished by their different speeds and polarizations. The linear evolution of the waves in a homogeneous medium is governed by the usual linear wave equations.

Consider a uniform, unperturbed, magnetic field $\mathbf{B}_0=(0,0,B_0)$ directed along the z axis, and the case of magnetosonic wave propagation across the field in the x direction (see Fig. 2). Then there are only fast magnetosonic waves (the slow wave is absent) that in the linear approximation is described by the equations:

$$\frac{\partial b_z}{\partial t}=-B_0\frac{\partial u_x}{\partial x}, \tag{8}$$

$$\rho_0\frac{\partial u_x}{\partial t}=-\frac{\partial}{\partial x}\left[c_s^2\rho+\frac{B_0 b_z}{4\pi}\right], \tag{9}$$

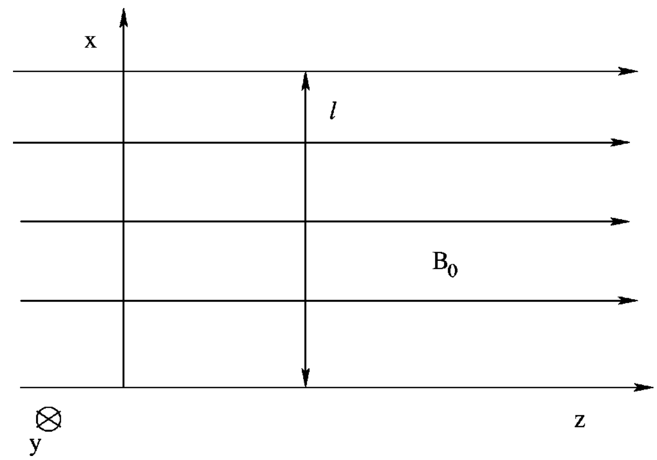

FIG. 2. The unperturbed magnetic field B_0 is directed along the z axis. The system is bounded in the x direction (l is the size of the system). Standing fast magnetosonic waves are polarized in the x direction, while Alfvén waves are polarized along the y axis and propagate along the z axis.

$$\frac{\partial \rho}{\partial t}=-\rho_0\frac{\partial u_x}{\partial x}, \tag{10}$$

where b_z and u_x are the perturbations of magnetic field and velocity, respectively, and $c_s=\sqrt{\gamma p_0/\rho_0}$ is the sound speed. Here and afterwards ρ denotes the perturbation of density [in Eqs. (4) and (5), ρ was the total density]. The wave equation for linear fast magnetosonic waves then follows

$$\frac{\partial^2 u_x}{\partial t^2}-V_f^2\frac{\partial^2 u_x}{\partial x^2}=0, \tag{11}$$

where $V_f=\sqrt{c_s^2+V_A^2}$ is the phase velocity of fast waves and $V_A=\sqrt{B_0^2/4\pi\rho_0}$ is the Alfvén speed.

The solution of the wave equation can be either propagating or standing patterns. The boundedness of the medium leads to the formation of a discrete spectrum of harmonics that represent the normal modes (eigenmodes) of the system. We consider the standing fast magnetosonic waves that have a straightforward extention to cylindrical (pulsating magnetic tube) and spherical (pulsating sphere with dipolelike magnetic field) geometries. The solutions for standing (plane) fast magnetosonic waves are

$$u_x=\alpha V_f\sin(\omega_n t)\sin(k_n x),$$

$$\rho=\alpha\rho_0\cos(\omega_n t)\cos(k_n x),$$

$$b_z=\alpha B_0\cos(\omega_n t)\cos(k_n x), \tag{12}$$

where $k_n=(n\pi/l)(n=1,2,\ldots)$ is the eigenvalue for a system of size l in the x direction, ω_n is the corresponding eigenfrequency, and α is the relative amplitude of the waves. Eigenvalues and eigenfrequencies are related by the dispersion relation $\omega_n/k_n=V_f$.

It is seen from the expressions (12) that standing fast magnetosonic waves cause a local periodical variation in both the density and the magnetic field. This variation is maximal near the nodes of the velocity and approaches to zero near the antinodes. The amplitude of the variation is considered to be small ($\alpha \ll 1$), and so does not affect the fast magnetosonic wave itself.

Consider now the influence of the density and the magnetic field variations (12) on Alfvén waves, considered to be polarized in the y-z plane. Then the velocity fields of fast magnetosonic and Alfvén waves are decoupled. The linear equations for Alfvén waves are

$$\frac{\partial b_y}{\partial t} = B_0 \frac{\partial u_y}{\partial z}, \tag{13}$$

$$\rho_0 \frac{\partial u_y}{\partial t} = \frac{B_0}{4\pi} \frac{\partial b_y}{\partial z}, \tag{14}$$

where b_y and u_y are small perturbations of the magnetic field and the velocity. These equations lead to the wave equation

$$\frac{\partial^2 b_y}{\partial t^2} - V_A^2 \frac{\partial^2 b_y}{\partial z^2} = 0. \tag{15}$$

The influence of the fast magnetosonic waves can be expressed by modifying Eqs. (13) and (14), which now became

$$\frac{\partial b_y}{\partial t} = (B_0 + b_z)\frac{\partial u_y}{\partial z} - \frac{\partial u_x}{\partial x} b_y, \tag{16}$$

$$(\rho_0 + \rho)\frac{\partial u_y}{\partial t} = \frac{B_0 + b_z}{4\pi} \frac{\partial b_y}{\partial z}. \tag{17}$$

Here we have neglected the advective terms $u_x \partial b_y / \partial x$ and $(\rho_0 + \rho) u_x \partial u_y / \partial x$ for several reasons. At the initial stage, the perturbations b_y and u_y of Alfvén waves propagating along the z axis do not depend on the x coordinate; each magnetic surface across x evolves independently. The x dependence arises due to the action of the fast magnetosonic waves, and so the neglected terms are second order in α^2. Moreover, we can consider the Alfvén waves at the velocity node of standing fast magnetosonic waves, where these terms are zero. In principle, the coordinate x stands as a parameter in Eqs. (16) and (17) of Alfvén waves.

Equations (16) and (17) lead to the Hill-type second-order differential equation

$$\frac{\partial^2 b_y}{\partial t^2} - \frac{(2B_0 + b_z)\dot{b}_z}{B_0(B_0 + b_z)} \frac{\partial b_y}{\partial t} - \frac{(B_0 + b_z)\ddot{b}_z - \dot{b}_z^2}{B_0(B_0 + b_z)} b_y - \frac{(B_0 + b_z)^2}{4\pi(\rho_0 + \rho)} \frac{\partial^2 b_y}{\partial z^2} = 0, \tag{18}$$

where $\dot{b}_z$ denotes the time derivative of the perturbing field. Introducing

$$b_y = h_y(z,t) \exp \int \frac{(2B_0 + b_z)\dot{b}_z}{2B_0(B_0 + b_z)} dt \tag{19}$$

and neglecting terms of order α^2 leads to the equation

$$\frac{\partial^2 h_y}{\partial t^2} - V_A^2 [1 + \alpha \cos(k_n x) \cos(\omega_n t)] \frac{\partial^2 h_y}{\partial z^2} = 0. \tag{20}$$

Comparing Eqs. (20) and (15) we can see that the influence of standing fast magnetosonic waves is expressed through a periodical variation of the Alfvén speed.

Performing a Fourier transform of h_y with $h_y = \int \hat{h}_y(k_z, t) e^{i k_z z} dk_z$, Eq. (20) leads to Mathieu's equation [20]

$$\frac{\partial^2 \hat{h}_y}{\partial t^2} + [V_A^2 k_z^2 + \delta \cos(\omega_n t)] \hat{h}_y = 0, \tag{21}$$

where

$$\delta = \alpha V_A^2 k_z^2 \cos(k_n x), \tag{22}$$

with x playing the role of a parameter. Equation (21) has main resonant solution if

$$\omega_A = \frac{B_0 k_z}{\sqrt{4\pi\rho_0}} = \frac{\omega_n}{2} \tag{23}$$

and it can be expressed as

$$\hat{h}_y = h_0 e^{(|\delta|/2\omega_n)t} \left[\cos\frac{\omega_n}{2} t - \sin\frac{\omega_n}{2} t \right], \tag{24}$$

where $h_0 = h(0)$. The solution has a resonant character within the frequency interval

$$\left| \omega_A - \frac{\omega_n}{2} \right| < \left| \frac{\delta}{\omega_n} \right|. \tag{25}$$

Equation (24) shows that the harmonics of Alfvén waves with half the frequency of fast magnetosonic waves growing exponentially in time. The growth rate of Alfvén waves is maximal at the velocity nodes of fast magnetosonic waves and tends to zero at the antinodes [see Eqs. (12) and (22)]. The amplitude of the magnetic field component in Alfvén waves depends on the x coordinate, i.e.. there is the periodical magnetic pressure gradient along this direction. Energy conservation implies that this gradient leads to the damping of initial fast magnetosonic waves, i.e., the energy transformed into Alfvén waves is extracted from fast magnetosonic waves. To show this, we consider the backreaction of amplified Alfvén waves on the initial fast magnetosonic waves.

The dependence of b_y on the x coordinate leads to an additional term in the equation of motion (9) for fast waves,

$$\rho_0 \frac{\partial u_x}{\partial t} = -\frac{\partial}{\partial x}\left[c_s^2 \rho + \frac{B_0 b_z}{4\pi} \right] - \frac{\partial}{\partial x}\left[\frac{b_y^2}{8\pi} \right]. \tag{26}$$

Therefore the wave equation (11) now becomes

$$\frac{\partial^2 u_x}{\partial t^2} - V_f^2 \frac{\partial^2 u_x}{\partial x^2} = -\frac{\partial^2}{\partial t \partial x}\left[\frac{b_y^2}{8\pi} \right]. \tag{27}$$

The additional term has the frequency of the initial fast magnetosonic waves ω_n (within the order of α^2) and can be considered as the external periodic force. At the initial stage it can be neglected as of second order. However, it becomes significant because of the exponential growth of amplitudes [see Eq. (24)]. It oscillates out of phase with respect to the initial fast waves (12), thus leading to their damping (as expected from physical considerations).

Note that Eqs. (1) and (2) describing the pendulum oscillations are similar to Eqs. (20) and (27) describing Alfvén waves and fast magnetosonic waves; the longitudinal oscillations of the pendulum correspond to the fast magnetosonic waves and the transversal oscillations correspond to the Alfvén waves.

Swing coupling between fast magnetosonic and Alfvén waves may be generalized from rectangular geometry to other symmetries, though a detailed description is beyond the scope of this paper.

IV. DISCUSSION

The suggested mechanism of energy transformation from fast magnetosonic into Alfvén waves has important consequences. It can be noted that the Alfvén waves hardly undergo either excitation or damping processes, while magnetosonic waves can be easily excited by external, even nonelectromagnetic, forces. Then swing interaction leads to the intriguing but natural suggestion that the energy of the nonelectromagnetic force that supports the magnetosonic waves in the system can be transmitted into the energy of purely magnetic incompressible oscillations. This result has many astrophysical applications. We briefly describe several of them.

A. Swing absorption

Resonant interaction between MHD waves, due to the inhomogeneity of the medium, was proposed by Ionson [8]. It arises where the frequency of an incoming wave matches the local frequency of the medium. Then a resonant energy transformation may take place, known as resonant absorption.

Similar phenomenon may also arise due to swing wave interaction. In this case fast magnetosonic waves can transform their energy into Alfvén waves, even in a homogeneous medium. For given medium parameters (magnetic field, density) the energy of fast waves may be "absorbed" by harmonics with wavelengths satisfying the resonant condition (23). Consequently, fast magnetosonic waves can transmit their energy into Alfvén waves in any spatial distribution of density or magnetic field. The process can be called *swing absorption*. The particular point of *swing absorption* is that energy absorption occurs through the harmonics with half the frequency of incoming waves [see Eq. (23)]. The process may be of importance in the earth's magnetosphere and in the solar atmosphere.

B. Torsional Alfvén waves in solar coronal loops

Swing wave interaction may play an important role in the excitation of torsional Alfvén waves in solar coronal loops. It may be suggested that any external action on the magnetic tube, anchored in the highly dynamical photosphere, causes a radial pulsation at the fundamental frequency, like a tuning fork (see also Ref. [21]). For a tube of radius r_0 the fundamental frequency of pulsation will be of the order V_f/r_0, where V_f is the phase velocity of fast magnetosonic waves at the photospheric level. If we consider the Alfvén and sound speeds to be of order $\sim 10\ \mathrm{km\,s^{-1}}$ and the radius of order $\sim 10^2$ km, then the period of fundamental mode of pulsation will be a few tens of seconds.

Radial pulsations of the tube may lead to the resonant (exponential) amplification of torsional Alfvén waves with half the frequency of the pulsations. These high-frequency torsional Alfvén waves can propagate upward and carry energy from the photosphere into the magnetically controlled corona or they may be damped in chromospheric regions leading to the heating of the chromospheric magnetic network.

C. Coupling between stellar pulsations and torsional oscillations

Swing wave interaction may be of importance in stellar interiors. A radial pulsation of a spherically symmetric star with dipolelike magnetic field may lead to the amplification of torsional oscillations. There are a number of energy sources that can support pulsations: radiation, nuclear reactions, tidal forces in binary stars, convection, etc. (e.g., Ref. [22]). Then the transformation of pulsational energy into torsional oscillations may lead to new sources for stellar magnetic activity.

V. CONCLUDING REMARKS

The swing wave-wave interaction [16] is developed here in the case of fast magnetosonic waves propagating across a magnetic field and Alfvén waves propagating along the field. In the case of oblique propagation, slow magnetosonic waves also exist and they may transmit their energy into Alfvén waves. In some cases the coupling between slow magnetosonic and Alfvén waves may be of importance. Also, the coupling in the case of different geometries (cylindrical, spherical) may be important in astrophysical situations. The most important result of swing wave interaction is that it reveals a new energy channel for Alfvén waves, permitting the transformation of energy of nonelectromagnetic origin into the energy of electromagnetic oscillations.

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